Benha University, Benha Faculty of Engineering Subject: Control Engineering (E451)	Time: 3hours, Forth Year 2013 Elect.Eng.Dept.

Solve as much as you can (questions in two pages)

Q1

a-Find the state space representation for the **field-controlled** DC motor?

[inputs are Vi, TL& outputs are speed ,torque & states are speed, current, angular displacement]

b- Find the state space representation for the **armature-controlled** DC motor?

[input is Vi & output is speed & states are speed, current]

Q2

Consider a control system has the following transfer function

$$\frac{Y(s)}{U(s)} = \frac{S^2 + S + 6}{S^3 + 2S^2 + 3S + 4}$$

a- Find the state space representation?

b- Draw the state space block diagram?

Q3

Consider a control system has a step input and the following state space representation

$$A = \begin{bmatrix} -7 & -10 \\ 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 1 \end{bmatrix}, D = [0]$$

a- Find the transition matrix? b- Find the states? **c-** Find the output?

d-Find the transfer matrix? e-Find the chara.equa.?

f- Find the eigen values? g-Find the closed loop poles?

h-Is the system complete controllable &observable?

i-Write a short MATLAB program to solve a,b,c,d,e,f,g.h?

Q4

a-Write and draw five controller arrangements?

b-Write six controller types depending on its control actions?

c-Write three compensator types depending on its function?

d-Explain the main functions of the P, PI, PD controllers?

Q5

An arrangement for controlling the viscosity of fuel oil is shown in Fig.1 in which heating is achieved by steam. A controller receives a feedback signal from the temperature sensor is used to control the steam throttle valve. The transfer functions of the main components are:

Controller = $G_c(s)$ Valve = $\frac{1}{S+1}$ heated process = $\frac{10}{S}$ sensor = 1

a- Draw the block diagram

b- Study the system (stability, step response) when

(i) $G_c(s) = K_p = 10$ (ii) $G_c(s) = K_p + K_I/S$

(iii) $G_c(s) = K_p + K_dS$ (iv) $G_c(s) = K_p + K_I/S + K_dS$

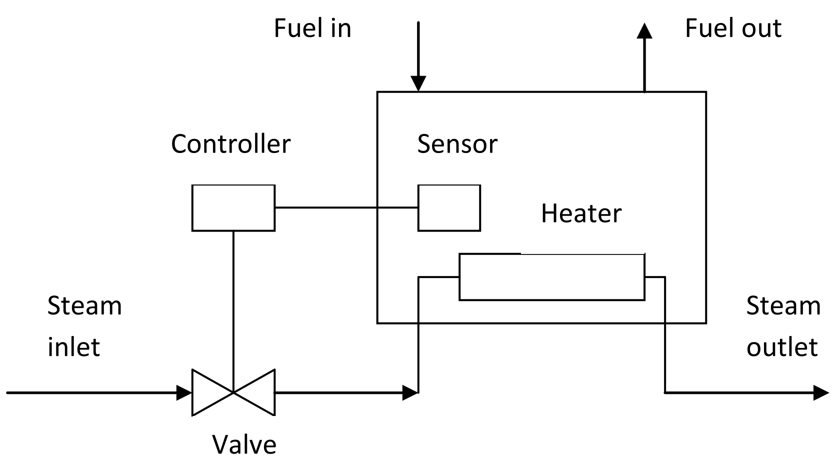


Fig. 1 controlling the viscosity of fuel oil

Q6

Consider a unity feedback control system has an open loop T.F=**[4/(S^2+2S)].** It has

$$\boldsymbol{A} = \begin{bmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{-4} & \mathbf{-2} \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} \boldsymbol{0} \\ \mathbf{1} \end{bmatrix}, \boldsymbol{C} = [\mathbf{4}\ \mathbf{0}], \boldsymbol{D} = [\mathbf{0}]$$

a-Find the state feedback controller (gain) using the pole placement method such that

the **new closed loop** transfer function is = **[9/(S^2+3.6S+9)]**?

b-Find the [$\mathbf{M_p}$, $\mathbf{t_s}$, $\mathbf{t_r}$, **&** $\mathbf{e_{ss}(t)}$] with and without the controller?

c- Write a short MATLAB program to solve a?

Model Answer

Q1

a-Find the state space representation for the **field-controlled** DC motor?

[inputs are Vi, TL& outputs are speed ,torque & states are speed, current, angular displacement]

-rotational motion $\sum \boldsymbol{T} = \mathbf{J}\ddot{\Theta}_{\mathbf{r}} = \mathbf{J}\dot{\boldsymbol{\omega}}_{\mathbf{r}} = \mathrm{T_m} - \mathbf{f}\boldsymbol{\omega}_{\mathbf{r}} - \mathrm{T_L}$

Where: J=moment of inertia, $\ddot{\Theta}_r$ =acceleration, T=torque

Kirchhoff's law; the algebraic sum of all voltages around a closed loop in an electrical circuit at any given instant is zero.

$$\textbf{loop voltage} = \sum_{1}^{n} \mathbf{V_{loop}} = \mathbf{0}, \quad \mathbf{V_f} = \mathbf{i_f R_f} + \mathbf{L_f}\frac{\mathbf{di_f}}{\mathbf{dt}}, \quad \mathbf{T_m} = \mathbf{K i_f} - -$$

$$\dot{X} = AX + BU \;, Y = CX + DU$$

$$\begin{bmatrix} \dot{\boldsymbol{\omega}}_{\mathbf{r}} \\ \dot{\mathbf{i}}_{\mathbf{f}} \\ \dot{\boldsymbol{\Theta}}_{\mathbf{r}} \end{bmatrix} = \begin{bmatrix} \frac{-f}{J} & \frac{K}{J} & 0 \\ 0 & \frac{-\mathbf{R_f}}{\mathbf{L_f}} & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{\omega}_{\mathbf{r}} \\ \mathbf{i_f} \\ \boldsymbol{\Theta} \end{bmatrix} + \begin{bmatrix} 0 & \frac{-1}{J} \\ \frac{1}{\mathbf{L_f}} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{V_f} \\ \mathbf{T_L} \end{bmatrix},$$

$$\begin{bmatrix} \boldsymbol{\omega}_{\mathbf{r}} \\ \mathbf{T} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & K & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{\omega}_{\mathbf{r}} \\ \mathbf{i_f} \\ \boldsymbol{\Theta} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{V_f} \\ \mathbf{T_L} \end{bmatrix},$$

b- Find the state space representation for the **armature-controlled** DC motor?

[input is Vi & output is speed & states are speed, current]

$$\dot{X} = AX + BU \;, Y = CX + DU$$

$$\begin{bmatrix} \dot{\boldsymbol{\omega}}_{\mathbf{r}} \\ \dot{\mathbf{i}}_{\mathbf{a}} \end{bmatrix} = \begin{bmatrix} \frac{-f}{J} & \frac{K}{J} \\ \frac{-\mathbf{K_b}}{\mathbf{L_a}} & \frac{-\mathbf{R_a}}{\mathbf{L_a}} \end{bmatrix} \begin{bmatrix} \boldsymbol{\omega}_{\mathbf{r}} \\ \mathbf{i_a} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{\mathbf{L_a}} \end{bmatrix} \mathbf{V_a} \quad , \boldsymbol{\omega}_{\mathbf{r}} = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{\omega}_{\mathbf{r}} \\ \mathbf{i_a} \end{bmatrix} + [0]\mathbf{V_a}$$

Qa- Find the state space representation?

$$\frac{Y(s)}{U(s)} = \frac{S^2 + S + 6}{S^3 + 2S^2 + 3S + 4}$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & -3 & -2 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, C = [6 \quad 1 \quad 1], D = [0]$$

b- Draw the state space block diagram?

Q3

Consider a control system has a step input and the following state space representation

$$\boldsymbol{A} = \begin{bmatrix} -7 & -10 \\ 1 & 0 \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \boldsymbol{C} = [1 \quad 1], \boldsymbol{D} = [0]$$

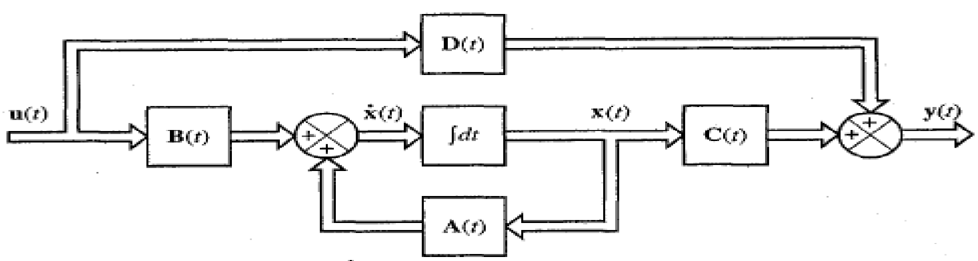
a- Find the transition matrix? b- Find the states? **c-** Find the output?

d-Find the transfer matrix? e-Find the chara.equa.?

f- Find the eigen values? g-Find the closed loop poles?

h-Is the system complete controllable &observable?

i-Write a short MATLAB program to solve a,b,c,d,e,f,g.h?



a- Find the transition matrix=$[SI-A]^{-1}$

$$[\boldsymbol{SI} - \boldsymbol{A}] = \begin{bmatrix} \boldsymbol{S+7} & \boldsymbol{10} \\ \boldsymbol{-1} & \boldsymbol{S} \end{bmatrix},$$

$$\boldsymbol{phis} = [\boldsymbol{SI} - \boldsymbol{A}]^{-1} = \begin{bmatrix} \boldsymbol{S+7} & \boldsymbol{10} \\ \boldsymbol{-1} & \boldsymbol{S} \end{bmatrix}^{-1} = \frac{\boldsymbol{1}}{\boldsymbol{S(S+7)+10}} \begin{bmatrix} \boldsymbol{S} & \boldsymbol{-10} \\ \boldsymbol{1} & \boldsymbol{S+7} \end{bmatrix}$$

Phi(t)=inverse Laplace of phi(s)

[(5*exp(-5*t))/3 - (2*exp(-2*t))/3, (10*exp(-5*t))/3 - (10*exp(-2*t))/3]

[exp(-2*t)/3 - exp(-5*t)/3, (5*exp(-2*t))/3 - (2*exp(-5*t))/3]

a- Find the states**=X(s)=phis*B*U(s)= xs =[1/(s^2 + 7*s + 10); 1/(s*(s^2 + 7*s + 10))]**

X(t) =inverse Laplace of x(s)= [exp(-2*t)/3 - exp(-5*t)/3; exp(-5*t)/15 - exp(-2*t)/6 + 1/10]

c- Find the output**=Y(s)=CX(s)+DU(s)=[1/(s^2 + 7*s + 10)+1/(s*(s^2 + 7*s + 10))]**

Y(t) =inverse Laplace of Y(s)= [exp(-2*t)/3 - exp(-5*t)/3+exp(-5*t)/15 - exp(-2*t)/6 + 1/10]

Find the transfer matrix=T(s)=C$[SI-A]^{-1}$B+D=**(s+1)/(s^2 + 7*s + 10)**

Find the chara.equa.**= det(SI-A)=(s^2 + 7*s + 10)**

Find the eigen values**=** the closed loop poles=roots(chara.equa.)**=roots(det(SI-A)**)=-2,-5

Is the system complete controllable &observable?

$$\boldsymbol{cont = [B \quad AB] = \begin{bmatrix} 1 & -7 \\ 0 & 1 \end{bmatrix}, |cont| = 1\ then\ complete\ cont.}$$

$$\boldsymbol{obsr = [C' \quad A'C'] = \begin{bmatrix} 1 & -6 \\ 1 & -10 \end{bmatrix}, |obsr| = -4\ then\ complete\ obser.}$$

Write a short MATLAB program to solve a,b,c,d,e,f,g.h?

a=[-7 -10;1 0];b=[1;0];c=[1 1];d=[0]; syms s;I=eye(2); phis=inv(s*I-a), ilaplace(phis), xs=phis*b*1/s, ilaplace(xs),Ys= c*XS, ilaplace(Ys),Ts=c*phis*b+d,

char=det(S*I-a), eig(a),roots(char),cont=ctrb(a,b),det(cont), obs=obsv(a,c), det(obs),

Q4

a-Write and draw five controller arrangements?

1-cascade arrangements 2-feedback arrangements

3-feedforward arrangements 4-state arrangements

5-compound arrangements

b-Write six controller types depending on its control actions?

Classifications of industrial controllers. Industrial controllers may be classified according to their control actions as:

1. Two-position or on–off controllers
2. Proportional controllers
3. Integral controllers
4. Proportional-plus-integral controllers
5. Proportional-plus-derivative controllers
6. Proportional-plus-integral-plus-derivative controllers

c-Write three compensator types depending on its function?

1-Lead compensator 2-Lag compensator 3-lag-Lead compensator

d-Explain the main functions of the P, PI, PD controllers?

1-the P controllers **reduces the rise time**

2-the PI controllers **reduces the steady state error**

3-the P controllers **reduces the maximum overshoot**

Q5-

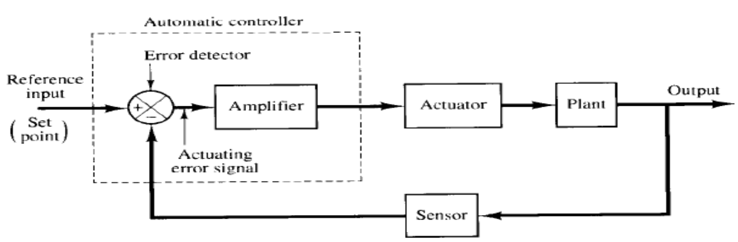
Amplifier=Controller = $G_c(s)$ actuator=Valve* heated process = $\frac{1}{S+1} * \frac{10}{S}$

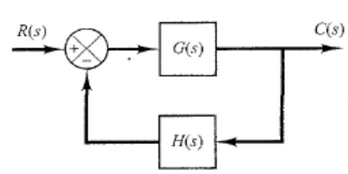
sensor = 1

b- Study the system (stability, step response) when

(i) $G_c(s) = K_p = 10$ (ii) $G_c(s) = K_p + K_I/S$

(iii) $G_c(s) = K_p + K_dS$ (iv) $G_c(s) = K_p + K_I/S + K_dS$





$G_c(s)\ G(s) = 10*10/[S(S+1)]$, $\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$, step=R(s)=1/s

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2},$$

Underdamped case $(0 < \zeta < 1)$: In this case, $C(s)/R(s)$ can be written

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{(s + \zeta\omega_n + j\omega_d)(s + \zeta\omega_n - j\omega_d)}$$

$$\begin{aligned} \mathcal{L}^{-1}[C(s)] &= c(t) \\ &= 1 - e^{-\zeta\omega_n t}\left(\cos\omega_d t + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t\right) \\ &= 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin\left(\omega_d t + \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}\right), \end{aligned}$$

C(s)=100/[s(s^2+s+100)], C(t)= (200*399^(1/2)*exp(-t/2)*sin((399^(1/2)*t)/2))/399

Q6

Consider a unity feedback control system has an open loop T.F=**[4/(S^2+2S)].** It has

$$\boldsymbol{A} = \begin{bmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{-4} & \mathbf{-2} \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} \boldsymbol{0} \\ \mathbf{1} \end{bmatrix}, \boldsymbol{C} = [\mathbf{4}\ \mathbf{0}], \boldsymbol{D} = [\mathbf{0}]$$

a-Find the state feedback controller (gain) using the pole placement method such that

the **new closed loop** transfer function is = **[9/(S^2+3.6S+9)]**?

b-Find the [**M_p, t_s, t_r, & $e_{ss}(t)$**] with and without the controller?

c- Write a short MATLAB program to solve a?

H=[h1 h2];det(sI-A+BH)

$$|\boldsymbol{SI} - \boldsymbol{A} + \boldsymbol{BH}| = \begin{vmatrix} \boldsymbol{s} & \mathbf{-1} \\ \mathbf{4} + \boldsymbol{h1} & \boldsymbol{s} + \mathbf{2} + \boldsymbol{h2} \end{vmatrix} = \boldsymbol{s}^2 + \boldsymbol{s}(\boldsymbol{h2} + \mathbf{2}) + \boldsymbol{h1} + \mathbf{4}$$

Then h1=9-4=5, h2=3.6-2=1.6

a=[0 1;-4 -2];b=[0;1];c=[4 0];c=[0];r=roots([1 3.6 9]); acker(a,b,r)

	$G1s=4/[s^2+4s]$ without	$G2s=9/[s^2+3.6s]$ with
$K_p = \lim_{s\to 0} G(s)$ $e_{ss} = \frac{1}{1 + K_p}$	$=\infty$ 0	$=\infty$ 0
$K_v = \lim_{s\to 0} sG(s)$ $e_{ss} = \frac{1}{K_v}$	1 1	9/3.6 3.6/9
$K_a = \lim_{s\to 0} s^2G(s)$ $e_{ss} = \frac{1}{K_a}$	0 $=\infty$	0 $=\infty$
$\sigma = \zeta\omega_n$		
$t_s = 4T = \frac{4}{\sigma} = \frac{4}{\zeta\omega_n}$ (2% criterion)		
$\beta = \tan^{-1}\frac{\omega_d}{\sigma}$		
$t_r = \frac{\pi - \beta}{\omega_d}$		
$\omega_d = \omega_n\sqrt{1 - \zeta^2}$		
$M_p = e^{-(\zeta/\sqrt{1-\zeta^2})\pi}$		
$\mathscr{L}^{-1}[C(s)] = c(t)$ $= 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin\left(\omega_d t + \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$		